Problem Set 3 - Solution - LV 141.A55 QISS

1. **Lindblad Equation**

$$\frac{d\rho}{dt} = \frac{-i}{\hbar}[H,\rho] + \sum_j L_j \rho L_j^\dagger - \frac{1}{2}\left\{L_j L_j^\dagger, \rho\right\}$$

$$\begin{aligned}\frac{d}{dt}\begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix} &= -i\omega_0\begin{pmatrix}0 & -\rho_{01}\\ \rho_{10} & 0\end{pmatrix} - i\omega_1\begin{pmatrix}\rho_{10}e^{-i\omega t} - \rho_{01}e^{i\omega t} & \rho_{11}e^{-i\omega t} - \rho_{00}e^{i\omega t}\\ \rho_{00}e^{i\omega t} - \rho_{11}e^{-i\omega t} & \rho_{01}e^{i\omega t} - \rho_{10}e^{-i\omega t}\end{pmatrix}\\ &+\gamma_r\begin{pmatrix}\rho_{11} & -\frac{1}{2}\rho_{01}\\ -\frac{1}{2}\rho_{10} & -\rho_{11}\end{pmatrix} + \gamma_\phi\begin{pmatrix}0 & -2\rho_{01}\\ -2\rho_{10} & 0\end{pmatrix}\end{aligned}$$

$$\mathbf{U_0} = \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$$\mathbf{U}_\mathrm{d} = \begin{pmatrix}0 & -e^{i\omega t} & e^{i\omega t} & 0\\ -e^{i\omega t} & 0 & 0 & e^{i\omega t}\\ e^{i\omega t} & 0 & 0 & e^{-i\omega t}\\ 0 & e^{i\omega t} & -e^{-i\omega t} & 0\end{pmatrix}$$

$$\mathbf{L}_r = \begin{pmatrix}0 & 0 & 0 & 1\\ 0 & -\frac{1}{2} & 0 & 0\\ 0 & 0 & -\frac{1}{2} & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$$

$$\mathbf{L}_\phi = \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & -2 & 0 & 0\\ 0 & 0 & -2 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

2. **Qubit Simulation - Relaxation**

```
from numpy import *
from scipy import integrate
from matplotlib.pyplot import *

def lindblad(t, rho):

#    parameters
   omega0 = 1     # qubit frequency
   gammar = 0.1   # relaxation
   gammap = 0     # dephasing
   omega  = 0     # driving frequency
   omega1 = 0     # driving amplitude

   U = array([
      [0,                          -omega1*exp(1j*omega*t), omega1*exp(-1j*omega*t),  0],
      [-omega1*exp(-1j*omega*t), -omega0,                  0,                        omega1*exp(
      [omega1*exp(1j*omega*t),   0,                        omega0,                   -omega1*exp
```

```
      [0,                          omega1*exp(1j*omega*t),  -omega1*exp(-1j*omega*t), 0]])

   Lr = array([
      [0,0,0,1],
      [0,-0.5,0,0],
      [0,0,-0.5,0],
      [0,0,0,-1]])

   Lp = array([
      [0, 0, 0,0],
      [0,-2, 0,0],
      [0, 0,-2,0],
      [0, 0, 0,0]])

   rho = transpose(rho)

   drhodt = -1j*dot(U,rho) + gammar*dot(Lr,rho) + gammap*dot(Lp,rho)

   return transpose(drhodt)

rho0 = array([0.5,0.5,0.5,0.5])

t =linspace(0,30,301)
dt = t[1]-t[0]
rho=zeros((4,len(t)),dtype=complex128)

r = integrate.ode(lindblad).set_integrator('zvode', method='bdf', with_jacobian=False)
r.set_initial_value(rho0, t[0])
for i,v in enumerate(t):
   if r.successful():
      r.integrate(r.t+dt)
      rho[:,i] = r.y
      #print r.t

sz     = real(rho[0,:]-rho[3,:])
trrho  = real(rho[0,:]+rho[3,:])
trrho2 = real(rho[0,:]**2+rho[3,:]**2+2*rho[1,:]*rho[2,:])

plot(t, sz, label='s_z')
plot(t, trrho, color='r', label='tr(rho)')
plot(t, trrho2, color='g', label='tr(rho^2)')

xlabel('t')
grid(True)
legend(loc='best')
savefig('relaxation.pdf')
show()
```

3. **Qubit Simulation - Dephasing**

   The decay of $\sqrt{\langle s_x\rangle^2 + \langle s_y\rangle^2}$ corresponds to $e^{-0.25t}$

4. **Qubit Simulation - Rabi oscillation**

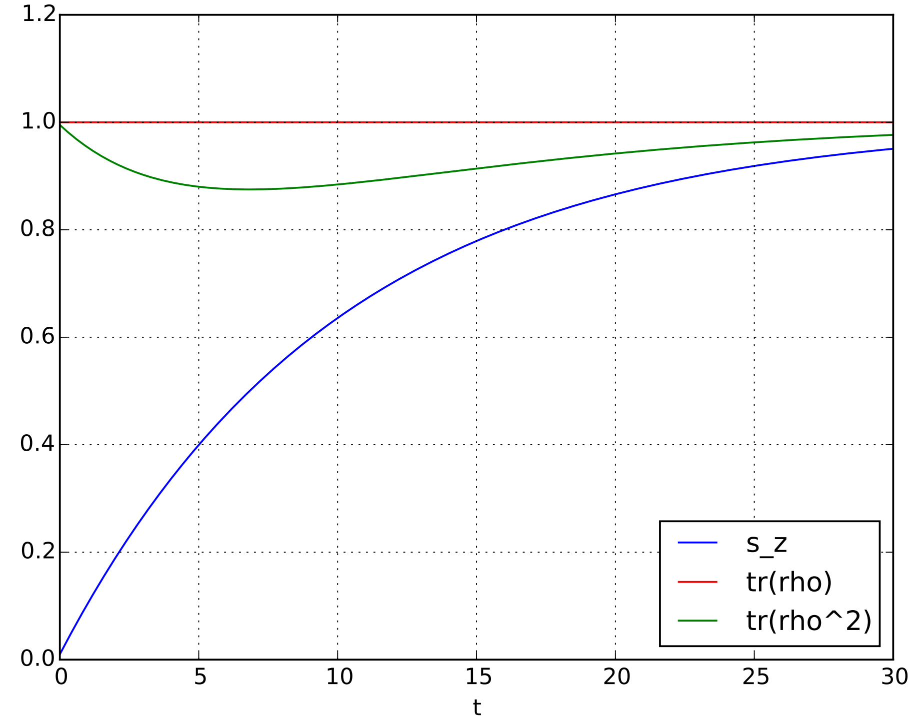

Figure 1: Qubit Simulation - Relaxation

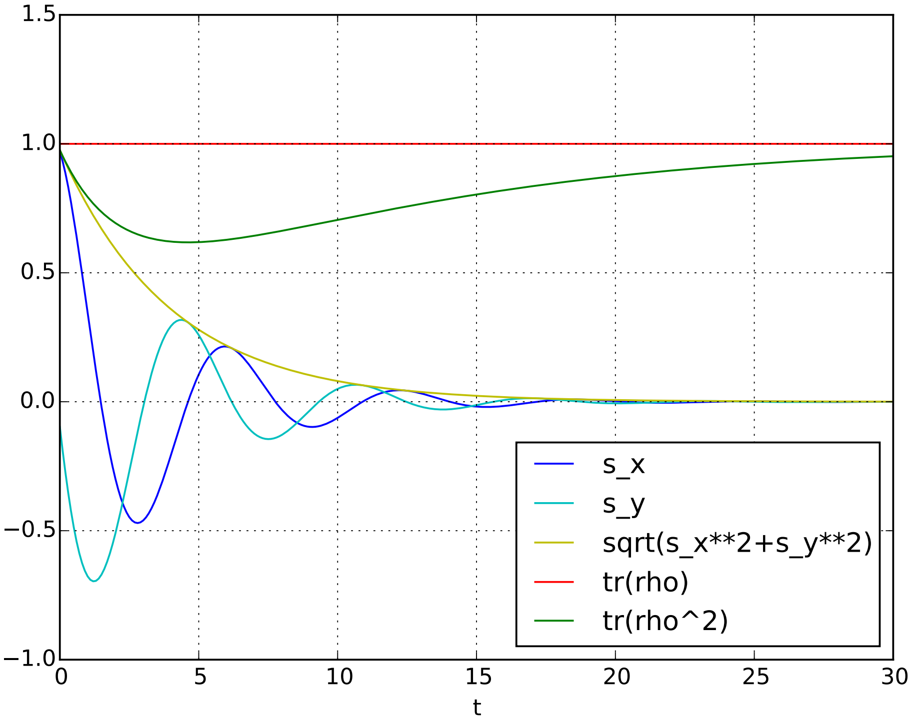

Figure 2: Qubit Simulation - Dephasing

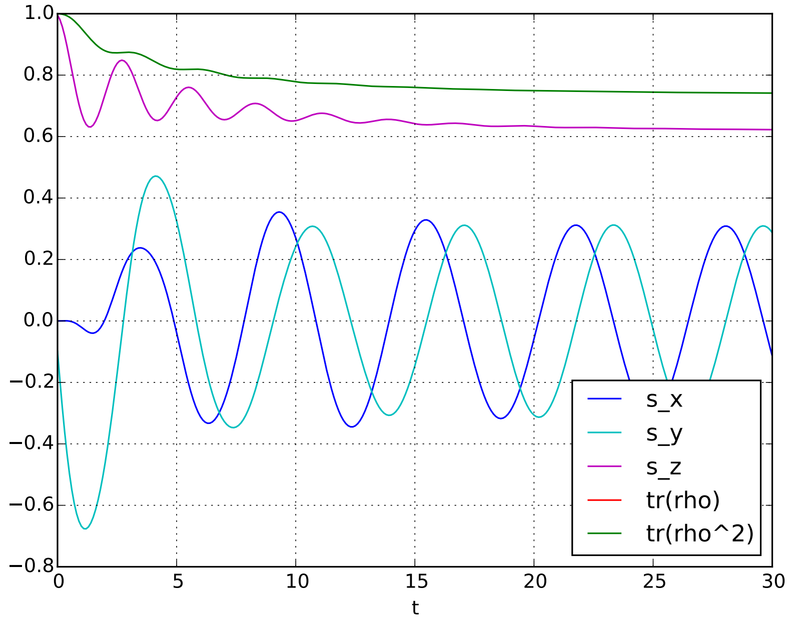

Figure 3: Qubit Simulation - Rabi oscillation with weak driving

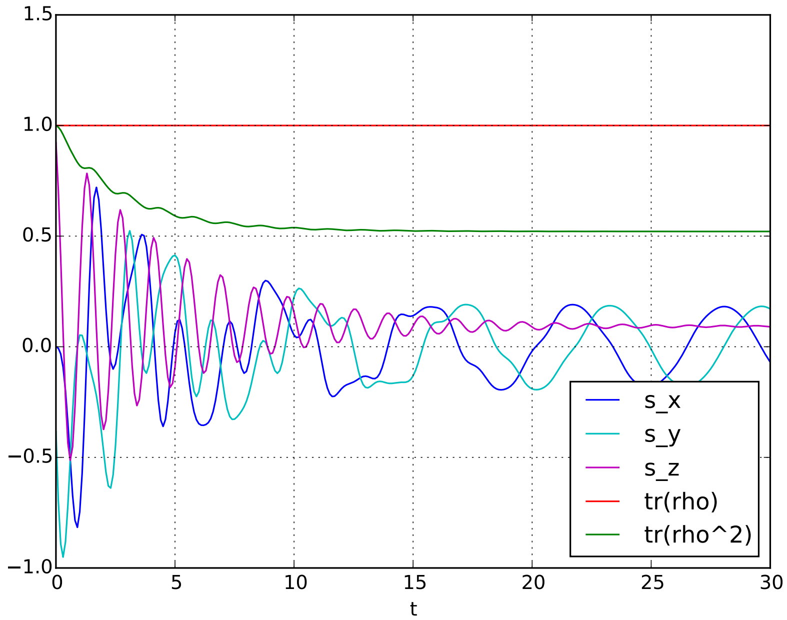

Figure 4: Qubit Simulation - Rabi oscillation with strong driving